Long-range correlations in quantum-chaotic spectra

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We discuss the long-range spectral correlations in random matrices. Their universality for one-band spectra and its breakdown for multiband spectra are investigated and characterized. The long-range properties are complementary to the usual short-range properties, and are important for conductance fluctuations in mesoscopic systems. However, unlike short-range properties, they are not ubiquitous in model quantum-chaotic systems. We formulate a system of multiply-kicked quantum rotors, and show that it exhibits both long-range and short-range correlations.

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I. INTRODUCTION

Random-matrix theory has had great success in explaining energy-level fluctuations in complex quantum systems. Typical applications include complex nuclei, atoms and molecules, model quantum-chaotic systems (e.g., billiards, coupled nonlinear oscillators, kicked rotors, and tops), microwave cavities, etc. [1–5]. More recently, attention has turned to glasses and amorphous clusters [6]. Another set of studies [7,8] has focused on conductance fluctuations in mesoscopic systems where random-matrix theory has again provided a strong basis for universality.

The study of energy-level fluctuations is based on quantities such as spacing distribution, number variance, etc. These involve correlations on the scale of the level spacing, and we shall refer to these properties as *local* or *short-range*. In contrast, conductance derives from the sum of all transmission eigenvalues and the corresponding fluctuations are *global* or *long-range* properties.

Many model quantum-chaotic systems are known to display short-range properties consistent with random-matrix theory. With regard to long-range properties, there have been studies focusing on conductance fluctuations in cavities [7,9] and kicked rotors [10]. To the best of our knowledge, there is no direct study of the long-range extensions of number variance and the two-point correlation function. In this article, we explore long-range spectral properties in random-matrix models and the extent of their universality. Further, we consider a model quantum-chaotic system where both short-range and long-range spectral correlations are *explicitly* realized. An important observation in this context is that long-range properties are not as universal as short-range properties.

This paper is organized as follows. In Sec. II, we discuss the long-range correlations in spectra of non-Gaussian random-matrix ensembles and their significance for universal conductance fluctuations (UCF). We focus on spectra which exhibit one-band and two-band structures and we illustrate our results via Monte Carlo (MC) simulations of ensembles with quartic potentials. In Sec. III, we discuss the long-range correlations for circular ensembles. In this context, we also discuss ensembles with weak periodic potentials. A system of multiply-kicked rotors is introduced in Sec. IV and shown to exhibit both short-range and long-range properties of circular ensembles. The well-known UCF result for single-band spectra is also confirmed. The results are summarized in Sec V.

II. NON-GAUSSIAN RANDOM-MATRIX ENSEMBLES

In this section, we consider random-matrix ensembles with the following probability density for eigenvalues $x_1, x_2, \ldots, x_N$ [11,12]:

$$P(x_1,\ldots,x_N) = c\prod_{j>k}|x_j - x_k|^{\beta}\prod_{l} e^{-\beta N V(x_l)}. \tag{1}$$

Here, c is the normalization constant, and N is the matrix dimension. The parameter β refers to the invariance properties of the ensembles with values 1, 2, and 4 denoting orthogonal, unitary, and symplectic cases, respectively. The weight function is defined in terms of the potential $V(x)$; the factor βN in the exponent results in the normalized eigenvalue density being independent of β and N [12]. In earlier studies [11,12], we have focused on the eigenvalue density and short-range fluctuations of the ensembles in (1) with *non-Gaussian* potentials.

The starting point for our study of long-range correlations is the two-point function [1]:

$$S_2(x,y) = \delta(x-y)R_1(x) + R_2(x,y) - R_1(x)R_1(y), \tag{2}$$

where R_n are n-level correlation functions:

$$R_n(x_1,\ldots,x_n) = \frac{N!}{(N-n)!}\int dx_{n+1}\ldots\int dx_N P(x_1,\ldots,x_N). \tag{3}$$

The physically relevant quantities for studies of long-range fluctuations are the moments of $S_2(x,y)$, defined as (with $p,q=0,1,2,\ldots$) [1]

$$C_{pq} = \int\int x^p y^q S_2(x,y)dx\,dy = \overline{M_p M_q} - \overline{M}_p\overline{M}_q. \tag{4}$$

Here, in the second step, $M_p = \mathrm{Tr}\, H^p$ with H being the corresponding random matrix, and the bar denotes an ensemble average over the probability density of $H, P(H) \propto \exp[-\beta N\,\mathrm{Tr}\,V(H)]$. A large class of potentials gives rise to level densities (R_1) with one-band structure [12] for $N\to\infty$.

Without loss of generality, we consider a single band with support $[-A,A]$. For such cases, a remarkable result emerges:

$$C_{pq} = \frac{2}{\beta}\left(\frac{A}{2}\right)^{p+q}\sum_{\zeta>0}{}' \zeta \begin{pmatrix} p \\ \frac{p-\zeta}{2} \end{pmatrix}\begin{pmatrix} q \\ \frac{q-\zeta}{2} \end{pmatrix}, \tag{5}$$

valid for $p+q$=even ($C_{pq}=0$ for $p+q$=odd). The sum in (5) is restricted to ζ such that $p-\zeta$=even. We have obtained this result directly from the polynomial method [11]. In analogy with the binary-correlation treatment of Gaussian ensembles [1,13], (5) can be interpreted as an expansion in the number (ζ) of H's correlated pairwise between the traces of H^p and H^q. We ignore $O(N^{-1})$ corrections in (5) and related results below. These are needed to describe short-range fluctuations. We also remark that the M_p become Gaussian variables for $p \ll N$.

Inverting the moments in (5), we obtain the corresponding two-point function:

$$S_2(x,y) = \frac{2}{\beta}g(x)g(y)\sum_{\zeta>0}\zeta v_\zeta(x)v_\zeta(y), \tag{6}$$

where $g(x)=(\pi A\sin\theta)^{-1}$, $x=A\cos\theta$, $0\leqslant\theta\leqslant\pi$, and $v_\zeta(x)=\cos(\zeta\theta)$. The sum in (6) is formally done using a cutoff and the result is

$$S_2(x,y) = -\frac{(A^2-xy)}{\beta\pi^2(x-y)^2\sqrt{(A^2-x^2)(A^2-y^2)}}. \tag{7}$$

The results in (5)–(7) were first obtained in the context of Gaussian ensembles $[V(x)=x^2/2]$ [1,13]. In recent work, (7) has been established for a wide class of non-Gaussian ensembles with one-band spectra [7,14–16].

Potentials with multiple minima (e.g., quartic potential, cosine potential, several infinite wells, etc.) admit the possibility of spectra with multiple bands [12,15]. Short-range fluctuations are not affected by the banding. However, for n-band spectra ($n>1$), (5) is no longer valid. The appropriate generalization consists of n branches corresponding to distinct values of $N(\bmod n)$. For illustration, consider the case of two symmetric bands with support $[-A,-B]$ and $[B,A]$. Using the labels $+/-$ for N=odd/even, respectively, we get the following result for $p+q$=even:

$$C^{\pm}_{pq} = \left[C^{\pm}_{11} - \frac{(A\pm B)^2}{2\beta}\right]\lambda_0^{p-1}\lambda_0^{q-1} + \frac{1}{\beta}\sum_{\zeta>0}{}'[(\zeta+1)\lambda^p_\zeta(\pm)\lambda^q_\zeta(\pm) + (\zeta-1)\lambda^p_\zeta(\mp)\lambda^q_\zeta(\mp)]. \tag{8}$$

Here, for p,ζ=even,

$$\lambda^p_\zeta(+) = \lambda^p_\zeta(-) = \frac{1}{\pi}\int_0^\pi x^p\cos\zeta\theta\, d\theta,$$

$$2x^2 = (A^2+B^2) + (A^2-B^2)\cos 2\theta, \tag{9}$$

while, for p,ζ=odd,

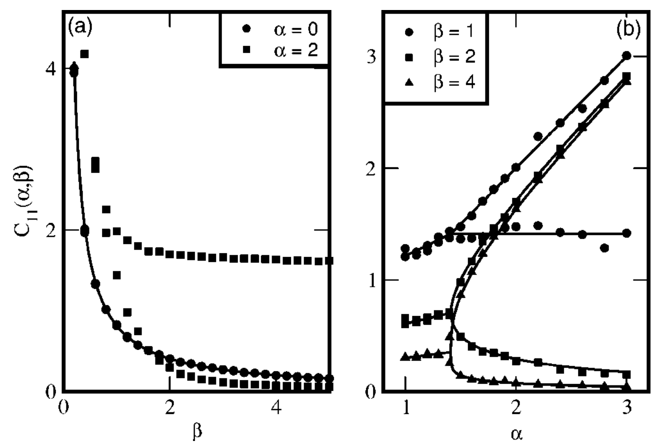

FIG. 1. MC results for C_{11} vs (a) β for $\alpha=0$ (one-band case), 2 (two-band case), and (b) α for $\beta=1,2,4$. For each (α,β) value, we use identical symbols for systems of size $N=200,201$. The solid lines denote analytical results for C_{11}; see discussion after (11).

$$2\lambda^p_\zeta(\pm) = (A\pm B)\lambda^{p-1}_{\zeta-1} + (A\mp B)\lambda^{p-1}_{\zeta+1}. \tag{10}$$

For $p+\zeta$=odd, $\lambda^p_\zeta(\pm)=0$. Inverting the moments as in (6) and summing the series, we obtain the corresponding two-point function [15]:

$$S^{\pm}_2(x,y) = -\frac{\epsilon(xy)}{\beta\pi^2\sqrt{(A^2-x^2)(x^2-B^2)(A^2-y^2)(y^2-B^2)}} \times\left[\frac{(A^2-xy)(xy-B^2)}{(x-y)^2} + \frac{A^2+B^2}{2} - \beta C^{\pm}_{11}\right], \tag{11}$$

where $\epsilon(t)=t/|t|$. For $\beta=2$, $C^{\pm}_{11}=(A\pm B)^2/4$. Our MC results discussed below suggest that $C^{\pm}_{11}=(A^2\pm B^2)/2$ for $\beta=1$ and $[(A+B\pm\sqrt{AB})^2-AB]/8$ for $\beta=4$. We stress that, for $B=0$ (one-band case), (8) reduces to (5) and (11) reduces to (7), since $C^{\pm}_{11}=A^2/2\beta$ in this limit.

In Ref. [12] we have described a MC procedure for generating non-Gaussian ensembles. We have used this method to study the level density and universality of short-range fluctuations for a large class of weight functions which exhibit multiband behavior. For long-range properties such as C_{pq}, we need to generate a much larger number of independent spectra than in our earlier work. We have done an extensive MC study of long-range correlations in ensembles with quartic potential $V(x)=(x^4-2\alpha x^2)/4$, which show a one-band→two-band transition in the spectrum at $\alpha=\sqrt{2}$; see Eqs. (14) and (15) of Ref. [12]. For the one-band case ($\alpha<\sqrt{2}$), the band parameter is $A=\sqrt{(2/3)[(\alpha^2+6)^{1/2}+\alpha]}$. For the two-band case ($\alpha>\sqrt{2}$), the band parameters are $A=\sqrt{\alpha+\sqrt{2}}$, $B=\sqrt{\alpha-\sqrt{2}}$. We have computed low-order C_{pq}'s and confirmed the validity of (5) and (8). Our studies were done for a wide range of (α,β) values. In each case, we generated 50 000 spectra for $N=200$, 201, and 10 000 spectra for $N=400,401$. These spectra were spaced apart by 100 MC steps.

Figures 1(a) and 1(b) plot C_{11} vs β for $\alpha=0$, 2, and C_{11} vs α for $\beta=1,2,4$—both for $N=200,201$. (The results for N

$=400,401$ are similar.) The solid lines superposed on the data sets are the above analytical expressions for C_{11}. To estimate sample errors in our calculations, we computed the autocorrelation function of the first moment M_1. The correlation decays to half its maximum value on a time scale which depends on α,β and is of the order of 100–500 MC steps. Therefore the relative sample error in Fig. 1 is about 1%.

Finally, let us discuss the relevance of the quantities C_{pq} in quantum transport problems. The conductance is proportional to $g=\Sigma T_j$ where $T_j \in [0,1]$ are eigenvalues of the $N\times N$ transmission matrix T, N being the number of channels. For large N, we identify $(H+A)/(2A)$ as a transmission matrix with eigenvalues $T_j=(x_j+A)/(2A)$. Then the variance of conductance fluctuations is

$$\mathrm{var}(g) = C_{11}/(4A^2), \tag{12}$$

yielding the universal result $(8\beta)^{-1}$ for one-band spectra [7,16]. One can similarly obtain variances of other physical quantities from (5). This universality breaks down at the onset of band-splitting, e.g., for the two-band case, conductance fluctuations become dependent on the ratio (B/A) and exhibit a strong odd-even effect with N [see (8) and the subsequent discussion]. The one-band result has been obtained earlier by different approaches and used to describe conductance fluctuations in quantum dots, e.g., chaotic cavities. However, the corresponding two-point correlation function (7) and the resultant number variance have not been explicitly demonstrated in model quantum-chaotic systems. To test these properties, we turn next to quantum maps and their random-matrix models.

III. CIRCULAR ENSEMBLES

We consider the (large-N) two-point correlation function for circular ensembles of unitary matrices, which are appropriate for quantum maps, and scattering (or transport) problems. In this case, the probability density for the eigenangles $(\theta_1,\theta_2,\ldots,\theta_N)$ is

$$P(\theta_1,\ldots,\theta_N) = c\prod_{j>k}|e^{i\theta_j}-e^{i\theta_k}|^{\beta}\prod_{l} e^{-\beta V(\theta_l)}, \tag{13}$$

where c is the appropriate normalization constant, and $V(\theta)$ is now a periodic potential. Let us first obtain the results for the potential-free case [$V(\theta)=0$], where the level-density $R_1(\theta)=N/(2\pi)$. In this case, Dyson has given correlation functions of all orders for finite N [3,17]. Using these results, we obtain the quantities C_{pq} and $S_2(\theta,\phi)$ correct to $O(N^{-1})$:

$$\begin{aligned} C_{pq} &\equiv \int_0^{2\pi}\int_0^{2\pi} e^{ip\theta}e^{iq\phi}S_2(\theta,\phi)d\theta\, d\phi \\ &= 2\beta^{-1}|p|\delta_{p+q,0}, \quad p,q=0,\pm1,\pm2,\ldots, \end{aligned} \tag{14}$$

$$S_2(\theta,\phi) = \frac{1}{2\pi^2\beta}\sum_{\zeta=-\infty}^{\infty}|\zeta|e^{i\zeta\psi} = -\frac{1}{4\pi^2\beta\sin^2(\psi/2)}, \tag{15}$$

where $\psi=\theta-\phi$. As before, the final expression in (15) is obtained by performing the sum with a cutoff.

We stress that (14) is valid for $|p|,|q| \ll N$; with larger $(|p|,|q|)$-values, C_{pq} (after suitable rescaling) gives the form factor for the short-range fluctuations. For calculation of the number variance below, we need the exact C_{pq}, which are nonzero only for $p+q=0$. For $\beta=2$, we have

$$\begin{aligned} C_{p,-p} &= |p|, \quad |p| \leqslant N, \\ &= N, \quad |p| \geqslant N. \end{aligned} \tag{16}$$

For $\beta=1$, we have

$$\begin{aligned} C_{p,-p} &= 2|p| - |p|\sum_{\mu=N'-|p|+1}^{N'}\frac{1}{\mu+|p|}, \quad |p|\leqslant N, \\ &= 2N - |p|\sum_{\mu=-N'}^{N'}\frac{1}{\mu+|p|}, \quad |p| \geqslant N, \end{aligned} \tag{17}$$

where $N'=(N-1)/2$. Finally, we have for $\beta=4$,

$$\begin{aligned} C_{p,-p} &= \frac{|p|}{2} + \frac{|p|}{4}\sum_{\mu=N-|p|+(1/2)}^{N-(1/2)}\frac{1}{\mu}, \quad |p|\leqslant 2N, \\ &= N, \quad |p| \geqslant 2N. \end{aligned} \tag{18}$$

A useful quantity to describe fluctuations is the number variance $\Sigma^2(r)$, viz, variance of the number of eigenangles in intervals of length $(2\pi r/N)$. In terms of C_{pq}, this is

$$\Sigma^2(r) = \sum_{p=-\infty}^{\infty} C_{p,-p}\frac{\sin^2(\pi pr/N)}{(\pi p)^2}. \tag{19}$$

Note that $\Sigma^2(r)=\Sigma^2(N-r)$. The short-range results are derived by replacing the sums in (19) by integrals, whereby the results become independent of N. However, for larger values of r, we need to deal with the sum directly. Using (16)–(18) in (19), we obtain large-N circular-ensemble results for $\Sigma^2(r)$ in compact forms, valid for both short and long range. We have, for $1 \lesssim r \lesssim N-1$,

$$\begin{aligned} \Sigma^2(r) &= \frac{1}{\pi^2}[\ln(\tilde r) + \gamma + 1], \quad \beta=2, \\ &= \frac{2}{\pi^2}\left[\ln(\tilde r) + \gamma + 1 - \frac{\pi^2}{8}\right], \quad \beta=1, \\ &= \frac{1}{2\pi^2}\left[\ln(2\tilde r) + \gamma + 1 + \frac{\pi^2}{8}\right], \quad \beta=4, \end{aligned} \tag{20}$$

where $\tilde r = 2N\sin(\pi r/N)$, and γ is the Euler constant. Note that $\tilde r=2\pi r$ for $r\ll N$. Thus, Eq. (20) generalizes the earlier short-range results [1].

The above results are also applicable to ensembles with $V(\theta)=O(1)$. [This should be contrasted with the case $V(\theta)=O(N)$, where spectra may appear in bands.] In these cases, the level density $R_1(\theta)=N/(2\pi)+O(1)$-corrections. For short-range correlations, the first term is adequate for the unfolding of the spectra. However, for long-range correlations, it turns out that the $O(1)$ correction is also needed to effect proper unfolding of the spectra. For $\beta=2$, we have explicitly computed the n-level correlation functions R_n using appropriate orthogonal polynomials on the unit circle. These exhibit (after proper unfolding) both long-range and short-range universalities. The corresponding calculation for $\beta=1,4$ requires skew-orthogonal polynomials on the unit circle. We have carried out MC studies of these ensembles for several potentials, and confirmed the universality of the number variance in (20). The details of this work will be published elsewhere [18].

IV. MULTIPLY-KICKED ROTORS

We now turn to quantum-chaotic systems where the long-range properties may be realized. We study systems in the kicked-rotor family [19,20] and clarify when long-range universality is realized. Consider quantum maps in an N-dimensional Hilbert space generated by the time-evolution operator U of a kicked rotor with torus boundary conditions. The standard case [19,20] is that of a singly-kicked rotor with $U=BG$, where $B\equiv B(\alpha)=\exp[-i\alpha\cos(\theta+\theta_0)/\hbar]$ and $G=\exp[-i(p+\gamma)^2/2\hbar]$ with θ,p being the position and momentum operators. Here, α is the kicking parameter, θ_0 is the parity-breaking parameter, and γ is the time-reversal-breaking parameter $(0\leqslant\gamma<1)$. In the position representation

$$B_{mn}=\exp\left[-i\frac{\alpha}{\hbar}\cos\left(\frac{2\pi m}{N}+\theta_0\right)\right]\delta_{mn}, \qquad (21)$$

$$G_{mn}=\frac{1}{N}\sum_{l=-N'}^{N'}\exp\left[-i\left(\frac{\hbar}{2}l^2-\gamma l-\frac{2\pi\mu l}{N}\right)\right], \qquad (22)$$

where $\mu=m-n, m,n=-N',-N'+1,\ldots,N',N'=(N-1)/2$ and we set $\hbar=1$. One knows [19,20] that, when parity is broken, $(\theta_0\neq 0)$ and N,α are sufficiently large (with $\alpha\gg N$); the eigenvalue spectrum of U accurately exhibits short-range random-matrix fluctuations (e.g., spacing distribution, number variance). These are characteristic of the $\beta=1$ case for $\gamma=0$ and $\beta=2$ case for $\gamma\neq 0$. We have confirmed this for samples of 50 000 matrices with $N=101{,}201$, $\theta_0=\pi/(2N)$, $\gamma=0.9$, where independent matrices are generated by setting $\alpha=j\alpha_0$ with $\alpha_0=20\,000$ and $j=1,2,\ldots,50\,000$. However, we find that the long-range quantities C_{pq} (for $|p|,|q|\ll N$) disagree with the circular-ensemble result in (14) (see Fig. 2). This is a direct consequence of the absence of the *uniformity principle* of periodic orbits (with low periods) in the corresponding semiclassical theory [20,21].

We therefore study a multiply-kicked rotor with M kicks: $U^{(2)}=B(\alpha_1)GB(\alpha_2)G\ldots B(\alpha_M)G$ where the α_k are spaced far apart; we choose $\alpha_k=(jM+k)\alpha_0$, where $k=1,\ldots,M$, and the index j labels independent spectra as above. We find that the universal long-range results for $\beta=2$ are realized when $M\geqslant 5$ for $\gamma=0.9$ (see Fig. 2 for $M=10$). It is interesting to note that $U^{(2)}$ for $\gamma=0$ also yields the $\beta=2$ results because the product is not symmetric. The $\beta=1$ results are recovered for the symmetric product $U^{(1)}=B(\alpha_1)G\ldots B(\alpha_M)GB(\alpha_{M-1})G\ldots B(\alpha_1)G$; Fig. 2 also shows results for this product with $M=10$. The short-range fluctuations are not affected by the product operations described above. This should be contrasted with the operator

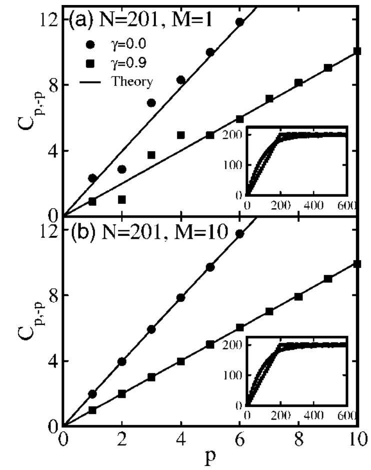

FIG. 2. $C_{p,-p}$ vs p for kicked-rotor spectra $(N=201,\gamma=0,0.9)$ for (a) $M=1$ and (b) $M=10$. The solid lines correspond to (14) for $\beta=1,2$. The insets show the corresponding data for $0\leqslant p\leqslant 600$, with solid lines denoting the analytical results in (16) and (17). Notice that the disagreement between data for the singly-kicked $(M=1)$ rotor and random-matrix predictions disappears at high values of p.

$U=(BG)^M$, which gives universal long-range properties but not the short-range properties.

For an accurate demonstration of the long-range properties in quantum-chaotic maps, we consider the number variance $\Sigma^2(r)$. For the kicked-rotor spectra, Σ^2 is calculated by considering all intervals $[2\pi k/N, 2\pi(k+r)/N \bmod (2\pi)]$ with $k=0,1,\ldots,N-1$. Figure 3 shows that the results for $M=10$ agree extremely well with the circular-ensemble predictions; the sample errors are very small ($\lesssim 0.5\%$ for $r\approx N/2$). The $M=1$ results show departures which become much larger and nonstationary (not shown in the figure) if only one value of k is chosen in the calculations. We have also analyzed the spectra for $M=3,5$; these show much smaller departures from the long-range theory than the $M=1$ case. Finally, we recall [3,22] that alternate eigenangles of the (even-

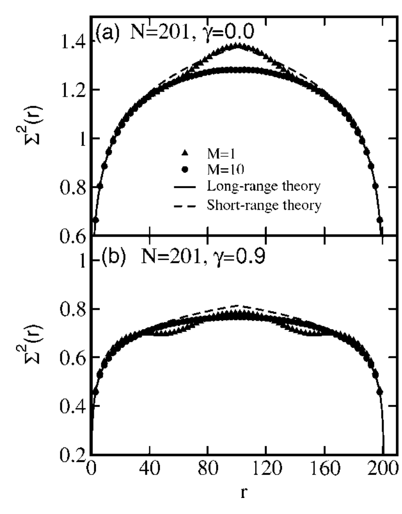

FIG. 3. $\Sigma^2(r)$ vs r for $N=201, M=1,10$, and (a) $\gamma=0$, (b) $\gamma=0.9$. The solid lines in (a), (b) correspond to the circular-ensemble result (20) with $\beta=1,2$ respectively. The dashed lines correspond to the short-range theory, viz, $\tilde{r}=2\pi r$ in (20).

dimensional) $\beta=1$ circular ensemble have the same statistics as that of the $\beta=4$ ensemble. We have confirmed the validity of this for multiply-kicked rotors.

It is natural to describe quantum transport via scattering matrices (say, U) which are modeled by circular ensembles. Then the conductance is given by $g=\Sigma_{m,n}|U_{mn}|^2$ with $m=1,2,\ldots,N_1$ and $n=N_1+1,\ldots,N$, where N_1 and $N_2(=N-N_1)$ are, respectively, the number of incoming and outgoing channels. The circular-ensemble prediction for its variance is (with $\eta=2/\beta$)

$$\mathrm{var}(g) = \frac{\eta N_1 N_2 (N_1 - 1 + \eta)(N_2 - 1 + \eta)}{(N-2+\eta)(N-1+2\eta)(N-1+\eta)^2}, \qquad (23)$$

which yields the above result $(8\beta)^{-1}$ for $N_1=N_2\gg 1$; see [7], and references therein. We have verified (23) for multiply-kicked rotor operators $G^{1/2}U^{(1)}G^{-1/2}$ and $U^{(2)}$ with both large and small number of channels N. However, for small values of N ($\geqslant 3$ for kicked rotors), the number of kicks needed for good agreement is higher (e.g., $M=50$). Our kicked-rotor calculations, along with earlier work [9,10], thus provide a firm basis for the utility of (23) in experiments on quantum dots.

V. CONCLUSION

We have studied the theory of long-range spectral fluctuations in random-matrix ensembles. We have elucidated the extent of universality for one-band and multiple-band spectra, and its relevance for conductance fluctuations. Further, we have explored the analogous long-range correlations in the eigenvalue spectra of circular ensembles and quantum-kicked rotors. We find that singly-kicked rotors do not exhibit universal long-range correlations. However, in multiply-kicked rotors, akin to multiple scattering in quantum transport problems, universal long-range correlations are recovered. In these systems, the circular-ensemble predictions for conductance fluctuations are explicitly verified for both the small and large number of channels.

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